Name:____ *Score:*_____/10

[3pt] **1.**

y

A. Sketch the graph of $f(x, y) = |x|$.

Solution:

B. Sketch the graph of $f(x, y) = -x^2 - 2y^2$. Also, draw the level curve of this function containing the point $(1, 2)$.

[3pt] **2.** The number of DVD players an online store sells per week is given by a function of two variables, $f(x, y)$, where x is the price per DVD player and y is the amount of money spent weekly on advertising. Suppose that the current price is \$ 150 per unit and that currently \$ 20 000 per week are being spent for advertising.

A. Would you expect $\frac{\partial f}{\partial x}(150, 20\,000)$ to be positive or negative? Why?

Solution:

When the price of the DVD player increases a small amount from \$ 150 and the amount of money spent on weekly advertising is fixed at \$ 20 000, the demand of the DVD player decreases. This means that while y is fixed at 20,000, f is a decreasing function of x. Hence $\frac{\partial f}{\partial x}(150, 20\,000) < 0$.

B. Would you expect $\frac{\partial f}{\partial y}(150, 20\,000)$ to be positive or negative? Why?

Solution:

When the price of the DVD player is fixed at \$ 150 and the amount of money spent on weekly advertising is increased a small amount from \$ 20 000, the demand of the DVD player increases. This means that while x is fixed at 150, f is an increasing function of y. Hence $\frac{\partial f}{\partial y}(150, 20\,000) < 0$.

[4pt] **3.** A farmer can produce $f(x, y) = 100\sqrt{2x^2 + y^2}$ units of produce by utilizing x units of labor and y units of capital.

A. Calculate the marginal productivities of labor and capital when $x = 10$ and $y = 5$.

Solution:

$$\frac{\partial f}{\partial y} = 100(1/2)(2x^2 + y^2)^{-1/2}(2y) = 100y(2x^2 + y^2)^{-1/2}$$

and

$$\frac{\partial f}{\partial x} = 100(1/2)(2x^2 + y^2)^{-1/2}(4x) = 200x(2x^2 + y^2)^{-1/2}.$$

The marginal productivity of labor when $x = 10$ and $y = 5$ is given by $\frac{\partial f}{\partial x}(10, 5) = 200(10)(2(10)^2 + 5^2)^{-1/2} = 400/3$.

The marginal productivity of capital when $x = 10$ and $y = 5$ is given by $\frac{\partial f}{\partial y}(10, 5) = 100(10)(2(10)^2 + 5^2)^{-1/2} = 200/3$.

B. Let h be a small number. What is the approximate effect on production of changing labor form 10 to $10 + h$ units while keeping capital fixed at 5 units?

Solution:

We have

$$f(10 + h, 5) - f(10, 5) \approx \frac{\partial f}{\partial x}(10, 5) \cdot h = 400h/3.$$

C. Estimate the change in production when labor decreases from 10 to 9.5 units while keeping capital fixed at 5 units.

Solution:

Using the above formula with $h = -1/2$, we have

$$f(9.5, 5) - f(10, 5) \approx -200/3.$$

D. Verify that

$$\frac{\partial^2 f}{\partial x \partial y}(10, 5) = \frac{\partial^2 f}{\partial y \partial x}(10, 5).$$

Solution:

We have

$$\frac{\partial^2 f}{\partial x \partial y} = (-1/2)100y(2x^2 + y^2)^{-3/2}(4x) = -200xy(2x^2 + y^2)^{-3/2}$$

and

$$\frac{\partial^2 f}{\partial y \partial x} = (-1/2)200x(2x^2 + y^2)^{-3/2}(2y) = -200xy(2x^2 + y^2)^{-3/2}.$$

So clearly $\frac{\partial^2 f}{\partial x \partial y}(10, 5) = \frac{\partial^2 f}{\partial y \partial x}(10, 5)$.