MATH 16B: ANALYTICAL GEOMETRY AND CALCULUS
SOLUTION FOR QUIZ #2

Problem 1. *Both first derivatives of the function* $f(x,y) = x^3 - 2xy + 4y$ *are zero at* $(2,6)$. *Use the second derivative test to determine the nature of* $f(x,y)$ *at* $(2,6)$. *If the second-derivative test is inconclusive, so state.*

Solution.

$$\frac{\partial f}{\partial x} = 3x^2 - 2y, \frac{\partial f}{\partial y} = -2x + 4$$
$$\frac{\partial^2 f}{\partial x^2} = 6x, \frac{\partial^2 f}{\partial x \partial y} = -2, \frac{\partial^2 f}{\partial y^2} = 0$$

So

$$D(x,y) = \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - (\frac{\partial^2 f}{\partial x \partial y})^2 = -4$$

Especially, $D(2,6) = -4 < 0$. So $f(x,y)$ is neither a relative maximum nor a relative minimum at $(2,6)$.

Problem 2. *Maximize the function* $x^2 - y^2$, *subject to the constraint* $2x + y - 3 = 0$. *(There are totally* 3 *versions. The other two versions are* $2x + y - 4 = 0$ *and* $2x + y - 5 = 0$ *respectively.)*

Solution. We need to solve the equations

$$\begin{cases} \frac{\frac{\partial(x^2-y^2)}{\partial x}}{\frac{\partial(x^2-y^2)}{\partial y}} = \frac{\frac{\partial(2x+y-3)}{\partial x}}{\frac{\partial(2x+y-3)}{\partial y}} \\ 2x + y - 3 = 0 \end{cases}$$

The first equation is

$$\frac{2x}{-2y} = \frac{2}{1}$$

So

$$x = -2y$$

Put it into the second equation, we get

$$-4y + y - 3 = 0$$
$$y = -1$$

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Then

$$x = 2$$

Therefore $x^2 - y^2$ gets maximum $2^2 - (-1)^2 = 3$ at $(2, -1)$, under the constraint $2x + y - 3 = 0$. (For constraint $2x + y - 4 = 0$, $x^2 - y^2$ gets maximum $\frac{16}{3}$ at $(\frac{8}{3}, -\frac{4}{3})$. For constraint $2x+y-5 = 0$, x^2-y^2 gets maximum $\frac{25}{3}$ at $(\frac{10}{3}, -\frac{5}{3})$.)

Problem 3. *Find the straight line that minimize the least-square error for the points* $(1, 8), (2, 4), (4, 3)$. *(The other two cases are* $(1, 8), (2, 4), (4, 5)$ *and* $(1, 8), (2, 3), (4, 5)$ *respectively.)*

Solution. Let the straight line be $y = Ax + B$. Then

$$E = (A + B - 8)^2 + (2A + B - 4)^2 + (4A + B - 3)^2$$

We need to solve equations

$$\frac{\partial E}{\partial A} = \frac{\partial E}{\partial B} = 0$$

Now

$$\frac{\partial E}{\partial A} = 2(A + B - 8) + 4(2A + B - 4) + 8(4A + B - 3) = 42A + 14B - 56$$

$$\frac{\partial E}{\partial B} = 2(A + B - 8) + 2(2A + B - 4) + 2(4A + B - 3) = 14A + 6B - 30$$

So the equations are

$$\begin{cases} 42A + 14B - 56 = 0 \\ 14A + 6B - 30 = 0 \end{cases}$$

Subtract three times of the second one from the first one, we get

$$-4B + 34 = 0$$

$$B = \frac{17}{2}$$

Then

$$A = \frac{30 - 6B}{14} = -\frac{3}{2}$$

Hence the straight line minimizing the least-square error for $(1, 8), (2, 4), (4, 3)$ is $y = -\frac{3}{2}x+\frac{17}{2}$. (For $(1, 8), (2, 4), (4, 5)$, the line is $y = -\frac{11}{14}+\frac{15}{2}$. For$(1, 8), (2, 3), (4, 5)$, the line is $y = -\frac{5}{7}x + 7$.)